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# Defocusing regimes of nonlinear waves in media with negative dispersion

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Defocusing regimes of quasimonochromatic waves governed by a nonlinear Schrödinger equation with mixed-sign dispersion are investigated. For a power-law nonlinearity, we show that localized solutions to this equation defined at the so-called *critical* dimension cannot collapse in finite time in the sense that their transverse (anomalously dispersing) and longitudinal (normally dispersing) extensions never vanish. Solutions defined at the *supercritical* dimension are proved to exhibit a nonvanishing mean longitudinal size, and cannot transversally collapse if they are assumed to shrink along each spatial direction.

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The nonlinear Schrödinger equation (NSE)

$$i\frac{\partial u}{\partial t}+\omega_{ij}\frac{\partial^2 u}{\partial x_i \partial x_j}+f(|u|^2)u=0, \tag{1}$$

$$u=u(x,t), \quad x\in \mathrm{R}^D, \quad i,j=1,\ldots,D,$$

is a generic equation which governs the dynamics of the envelope $u(x,t)$ of weakly nonlinear dispersive waves that propagate in a comoving frame with the group velocity $v_g=|\mathbf{v}_g|=(\partial\omega/\partial k)(\mathbf{k}_0)$ and with the dispersion coefficients $\omega_{ij}=\frac{1}{2}(\partial^2\omega/\partial k_i \partial k_j)(\mathbf{k}_0)$. Here, $\omega(\mathbf{k}_0)$ and $\mathbf{k}_0$ are the frequency and the wave vector of the carrier wave, respectively, $D$ denotes the number of space dimensions, and the nonlinear frequency shift $\Delta\omega\equiv f(|u|^2)$ in Eq. (1) is assumed to depend locally on the wave intensity $|u|^2$. By convention, repeated subscripts imply summation. Equation (1) arises in a wide variety of contexts, such as nonlinear optics and plasma physics [1–4], when the characteristic time of the intensity variation remains greater than the characteristic relaxation time of the low-frequency fluctuations induced by the high-frequency carrier wave. For example, for an optical Kerr medium, the nonlinear term of Eq. (1), related to the intensity-dependent refractive index of the medium, reduces to $f(|u|^2)\propto|u|^2$. In isotropic media $[\omega=\omega(|\mathbf{k}|)]$, the tensor $\omega_{ij}$ can be represented in the form

$$\omega_{ij}=(v_g/2k)(\delta_{ij}-k_ik_j/k^2)+\omega'' k_ik_j/(2k^2), \tag{2}$$

where $\omega''=\partial^2\omega/\partial k^2|_{\mathbf{k}_0}$. The first term in this expression is responsible for the wave diffraction in the transverse plane orthogonal to the carrier wave vector $\mathbf{k}_0\|\mathbf{v}_g$. Usually, this term is positive definite and yields the transverse Laplacian in Eq. (1). The second term in (2) models the dispersive broadening of the wave packet and may exhibit both positive and negative signs: in nonlinear optics, the case $\omega''>0$ corresponds to the so-called *anomalous* dispersion, and $\omega''<0$ to the *normal* dispersion. In this context, the space-time variables $z$ ($\hat{z}\|\mathbf{k}_0$) and $t$ are often inverted: when looking at structures evolving in the group velocity frame $t\to t-z/v_g$, $z\to z$, the variable $t$ in (1) refers to the propagation variable, while $z$ plays the role of a retarded time [2]. In addition, Eq. (1) also generally governs the propagation of nonlinear waves in anisotropic media, as is the case of self-focusing waves in magnetized plasmas [3]. When transforming the operator $L=\omega_{ij}(\partial^2/\partial x_i \partial x_j)$ into a canonical form by a rotation of the coordinate system and rescaling the variables and field, Eq. (1) reads

$$i\partial_t u+g^{jk}\partial_j\partial_k u+f(|u|^2)u=0. \tag{3}$$

In Eq. (3), the new dispersion operator $g^{jk}\partial_j\partial_k\equiv\nabla_\perp^2+s\nabla_z^2$ ($\partial_j\equiv\partial/\partial x^j$) contains a diagonal metric tensor of components $g^{jk}$, with $D_\perp$ elements equal to $+1$, corresponding to the *transverse* space vector $\mathbf{r}_\perp$, and with $D_z$ elements of value $s=\pm1$, corresponding to the *longitudinal* one $\mathbf{z}$, so that $D=D_\perp+D_z$.

The nature of the nonlinear interaction significantly depends on the signs of both the dispersion coefficient $s$ and the nonlinear frequency shift $f(|u|^2)$. For $s=+1$, Eq. (1) is called the *elliptic* nonlinear Schrödinger equation (ENSE) and possesses blowing-up solutions for certain classes of initial data $u(x,0)$: as reviewed in [5], blowup may occur when the potential $U=-f(|u|^2)$ in (3) is negative and obeys the requirement

$$(D+2)\int F(|u|^2)d^Dx\leqslant D\int f(|u|^2)|u|^2d^Dx \tag{4}$$

with $F(v)=\int_0^v f(w)dw$. Henceforth assuming this, a sufficient condition for blowup is that the Hamiltonian integral

$$H=\int \partial_j u g^{jk}\partial_k u^* d^Dx-\int F(|u|^2)d^Dx, \tag{5}$$

which is a constant of motion for Eq. (3), has to be negative. As is well known, the mathematical proof of a finite-time blowup results from the vanishing of the mean square radius—or virial—integral $I(t)=\int x^2|u|^2d^Dx$ at a finite time $t=t_c$, leading thereby to the divergence of the gradient norm together with the divergence of the amplitude $|u|$ as $t\to t_c$. One has, however, to remember that the time at which $|u|\to+\infty$ can be smaller than $t_c$, which indicates that

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blowup occurs *before* $I(t)$ reaches zero. Conversely, a nonvanishing $I(t)$ does not, strictly speaking, guarantee the absence of blowup. Nevertheless, the vanishing of $I(t)$ in the context of ENSE has above all a physical meaning: it emphasizes the local concentration of the intensity at the center, which is responsible for the occurrence of a singularity and thus promotes the *collapse* of localized wave fields. Collapse is then connected with the attractive character of the self-interaction of waves along each spatial direction. So, from a physical viewpoint, the behavior of the virial integral is more meaningful than a mathematical argument showing—or not—the existence of blowup. In the following, we will hence regard the nonvanishing of virial-type integrals as being a signature of the absence of a "physical" collapse in the sense given above, even though it cannot definitively settle the question of blowup. Among the nonlinear forms for which the collapse is possible, we can recall the general power nonlinearity $f(|u|^2)=|u|^{2\sigma}$, $\sigma>0$, which includes the cubic NSE for $\sigma=1$, and for which negative-energy states of ENSE self-focus and blow up in finite time when $D$ is larger than or equal to the so-called *critical* value $2/\sigma$.

When $s=-1$, Eq. (1) is called the *hyperbolic* nonlinear Schrödinger equation (HNSE) and the nature of the nonlinear interaction deeply differs from the elliptic case, in the sense that a compression of the wave form in the transverse plane may be counteracted by a repulsion along the $z$ axis. During the last decade, this problem was a subject of intensive study [2–9]. Nevertheless, the description of such nonlinear regimes is basically an open problem, since apart from these studies, to our knowledge no thorough investigations of the global behavior of multidimensional solutions to HNSE are available so far. The present work is devoted to establishing time-dependent estimates governing the evolution of $u(x,t)$, when the solution, assumed to exist at least locally in time, evolves from an initially localized-in-space function. By means of two virial-type identities, the time evolutions of localized wave packets governed by HNSE are described at both the *critical* ($D=2/\sigma$) and *supercritical* ($D>2/\sigma$) dimensions for a power nonlinearity. We show that no collapse can occur at the critical dimension and that the longitudinal extension of the solution never reaches zero for any $D$. Moreover, in the supercritical case, we demonstrate that a transverse self-focusing cannot be achieved by a collapse when assuming that the mean longitudinal size of $u(x,t)$ decreases in time.

Before proceeding, we prove that nonzero localized stationary solutions of (3), defined as $u(x,t)=\phi(x)e^{i\lambda t}$, cannot exist for $s=-1$. We argue by contradiction and suppose that $\phi(x)$ is a nonzero localized solution that satisfies the differential equation

$$-\lambda\phi+\nabla_\perp^2\phi-\nabla_z^2\phi+f(|\phi|^2)\phi=0. \tag{6}$$

We multiply (6) by $(\mathbf{r}_\perp\cdot\nabla_\perp\phi^*)$ and by $(\mathbf{z}\cdot\nabla_z\phi^*)$, then integrate the real part of the resulting equations to get

$$\lambda\|\phi\|_2^2+(1-2/D_\perp)\|\nabla_\perp\phi\|_2^2-\|\nabla_z\phi\|_2^2-\int F(|\phi|^2)d^Dx=0, \tag{7}$$

$$\lambda\|\phi\|_2^2+\|\nabla_\perp\phi\|_2^2-(1-2/D_z)\|\nabla_z\phi\|_2^2-\int F(|\phi|^2)d^Dx=0, \tag{8}$$

where $\|g\|_p=(\int|g|^pd^Dx)^{1/p}$ denotes the standard $L^p$ norm. Subtracting Eq. (8) from Eq. (7) gives $\|\nabla_\perp\phi\|_2^2=-(D_\perp/D_z)\|\nabla_z\phi\|_2^2$, which is incompatible with the positiveness of norms. The same proof can be extended to traveling wave solutions (see the recent paper [6]). Even if it displays that the most "simple" solutions to HNSE exhibit a natural property of *unlocalized* structures, this result does not prevent *time-dependent* solutions $u(x,t)$ from remaining localized, at least in a finite time interval. In order to follow the time evolution of the spatial extensions of $u(x,t)$, let us introduce two quantities, denoted by $I_\perp(t)\equiv\int r_\perp^2|u|^2d^Dx$ and $I_z(t)\equiv\int z^2|u|^2d^Dx$, respectively, which, normalized by the mass $N=\int|u|^2dx$, represent the transverse and longitudinal mean square radius of a localized structure. Straightforward calculations, following [10,5], lead to two virial-type identities,

$$\ddot{I}_\perp(t)=4\left\{2\|\nabla_\perp u\|_2^2+D_\perp\int[F(|u|^2)-|u|^2f(|u|^2)]d^Dx\right\}, \tag{9}$$

$$\ddot{I}_z(t)=4\left\{2\|\nabla_z u\|_2^2-D_z\int[F(|u|^2)-|u|^2f(|u|^2)]d^Dx\right\}. \tag{10}$$

For a power-law nonlinearity $f(|u|^2)=|u|^{2\sigma}$ ($\sigma>0$), the nonlinear potential integrals in Eqs. (9) and (10) simply reduce to

$$\int[F(|u|^2)-|u|^2f(|u|^2)]d^Dx=-\frac{\sigma}{\sigma+1}\|u\|_{2(\sigma+1)}^{2(\sigma+1)}. \tag{11}$$

To describe the evolution of $I_\perp(t)$ and $I_z(t)$, we now integrate by parts the $L^2$ norm of any $L^2$-integrable function $g$ and apply the Schwarz inequality to obtain the key inequality

$$\|g\|_2^2\leqslant\frac{2}{D_i}\|\nabla_i g\|_2\|x^i g\|_2,\quad i=(\perp,z), \tag{12}$$

which resembles the Heisenberg uncertainty relations, when it is normalized by the "mass" $\|g\|_2^2$. Then, decomposing the wave field $u(x,t)$ as $u(x,t)=A(x,t)e^{i\varphi(x,t)}$, we estimate separately each contribution of the gradient norm

$$\|\nabla_i u\|_2^2=\|\nabla_i A\|_2^2+\|A\nabla_i\varphi\|_2^2. \tag{13}$$

Applying the inequality (12) to the first term of the right-hand side (RHS) of (13) immediately gives $\|\nabla_i A\|_2^2\geqslant(D_i/2)^2N^2/I_i$ with $I_i(t)\equiv\int x_i^2|u|^2d^Dx$. The last term of (13) can be bounded as $\|A\nabla_i\varphi\|_2^2\geqslant(\dot{I}_i)^2/(16I_i)$. This estimation follows after using the identity $\partial_t I_i(t)=4\epsilon_i\int x^iA^2\nabla_i\varphi d^Dx$ with $\epsilon_\perp=1$ and $\epsilon_z=-1$ and applying the Schwarz inequality to this integral. By so doing, we finally get the bound of the gradient norm from below,

$$\|\nabla_i u\|_2^2\geqslant(D_i/2)^2N^2/I_i+(\dot{I}_i)^2/(16I_i) \tag{14}$$

(see a similar treatment in [11]). We now prove that under the requirement (4), the vanishing of the longitudinal extension of $u(x,t)$ along the $z$ direction is not possible.

*(a) Absence of a longitudinal collapse for all space dimensions.* First, we notice that since $I_z(t)$ is positive, the vanishing of this integral at a hypothetical time $t_c$ would necessarily imply that $I_z(t)$ should decrease near $t_c$, otherwise the complementary situation $\dot{I}_z \geq 0$ would immediately lead to the wanted result. For treating the relevant case $\dot{I}_z < 0$, we retain the estimate

$$\ddot{I}_z > 8\|\nabla_z u\|_2^2 \tag{15}$$

from Eq. (10), where the nonlinear integrals form a positive contribution by virtue of (4), and apply the bound (14) to get

$$\ddot{B} > -\frac{\partial}{\partial B}\mathcal{U}(B), \quad \mathcal{U}(B) \equiv \frac{C^2}{2B^2}, \tag{16}$$

with $B(t) \equiv \sqrt{I_z(t)}$ and $C = -D_z N$. Inequality (16) describes the motion of a particle moving under the action of a potential whose effective force $-\partial\mathcal{U}/\partial B$ is stronger than $C^2/B^3$. Such a repulsive potential pushes the particle out from the center $B=0$ towards large values of $B$ and hereby prevents the particle from reaching the value $B=0$. To show the absence of a longitudinal collapse, we multiply both sides of (16) by $\dot{B}<0$ (since $\dot{I}_z<0$) and obtain after a simple integration over time:

$$\dot{B}^2 + C^2/B^2 \leq \mathcal{E}(0) \equiv [\dot{B}(0)]^2 + C^2/[B(0)]^2, \tag{17}$$

where the "initial" instant $t=0$ here refers to the moment when $I_z$ starts to decrease. As $\mathcal{E}(0)$ remains finite, the limit $B \to 0$ is strictly forbidden in that case. The minimum extension of the solution $u(x,t)$ along the axis of normal dispersion is then given by $B_{min}^2 = I_{z,min} = C^2/\mathcal{E}(0)$. As $I_z(t)$ never vanishes, we can multiply (15) by $I_z$ and apply the inequality (12) with $g=u$ to get $(d^2/dt^2)(I_z^2) \geq 2I_z\ddot{I}_z > 4C^2$, implying a spreading of $I_z(t) \geq I_z^*(t) \sim t$ in the limit $t \to +\infty$. This dynamics indicates that $I_z(t)$ must finally increase in an extension regime $\dot{I}_z > 0$ for which the estimate (16) strengthens the asymptotic divergence of $I_z(t)$ with $I_z(t) > I_z^*(t) \sim t^2$ as $t \to +\infty$ since $B(t) > \dot{B}(0)t + B(0)$. The behavior of the characteristic longitudinal size $\sqrt{I_z}$ of the wave field is thus to grow at least linearly in time. Note that the previous result holds independently of the space dimension $D$. This means that the spatial distribution of the wave field tends to displace in the longitudinal direction, either by dispersing simply along the $z$ axis, or by moving the maximum of $u(x,t)$ from the origin $z=0$ towards larger distances $z$, as observed in the numerical simulations of Refs. [8,9].

Let us now study the behavior of the transverse mean square radius $I_\perp(t)$. As the dynamics of this latter integral essentially depends on the nonlinear frequency shift $\Delta\omega$, we will restrain our investigation to power-law nonlinearities characterized by the potential contribution (11) with a transverse dimension number satisfying $D_\perp \leq 2/\sigma$. This assumption holds in the situations of physical interest $D_\perp = 2$, $D_z = 1$, regarding the propagation of short optical pulses in normally dispersive Kerr media with a cubic nonlinearity $\sigma=1$. We investigate the critical and the supercritical dimensions, separately.

*(b) Absence of transverse collapse at the critical dimension $D=2/\sigma$.* Referring to the standard two-dimensional (2D) situation $\sigma=1$, we here assume $D_\perp = D_z = 1/\sigma$. In this case, we prove that $I_\perp(t)$ never vanishes: first, one sees that by using (11), Eq. (9) reads

$$\begin{aligned}\ddot{I}_\perp(t) &= 8\left[H + \|\nabla_z u\|_2^2 + \frac{1-\sigma D_\perp/2}{\sigma+1}\|u\|_{2(\sigma+1)}^{2(\sigma+1)}\right] \\ &> 4\left[H + \|\nabla_\perp u\|_2^2 + \frac{1-\sigma D_\perp}{\sigma+1}\|u\|_{2(\sigma+1)}^{2(\sigma+1)}\right] \\ &= 4[H + \|\nabla_\perp u\|_2^2].\end{aligned} \tag{18}$$

Bounding the transverse gradient norm by employing (14) in the RHS of (18), we easily find

$$\ddot{X} > 3HX^{-1/3} + (3/4)D_\perp^2 N^2 X^{-5/3} \tag{19}$$

with $X(t) \equiv [I_\perp(t)]^{3/4}$. When supposing *a priori* that $I_\perp(t)$ could possibly vanish at a given time $t_c$, this transverse virial integral should necessarily decrease as $t \to t_c$. Eluding thus the trivial case $\dot{X}>0$ that cannot promote a transverse collapse, we just consider a decreasing integral $X(t)$ and multiply the estimate (19) by $\dot{X}<0$ to find

$$\dot{X}^2 - 9HX^{2/3} + (9/4)D_\perp^2 N^2 X^{-2/3} \leq \mathcal{E}'(0). \tag{20}$$

Similarly to the previous case, the constant $\mathcal{E}'(0)$ is defined by the left-hand side (LHS) of the above inequality stated at the "initial" moment $t=0$ at which $I_\perp(t)$ begins to decrease. Since this constant is ensured to be finite, one sees from the boundedness of (20) that it is impossible to pass to the limit $X(t) \to 0$, which concludes the proof. Furthermore, as $I_\perp$ never vanishes, the transverse size of $u(x,t)$ evolves asymptotically faster or slower than the longitudinal one depending on the sign of $H$. In the critical case $D=2/\sigma$, relations (9) and (10) can be combined into the simple form

$$\ddot{I}_\perp - \ddot{I}_z = 8H. \tag{21}$$

A direct integration of (21) then yields $I_\perp(t) = I_z(t) + 4Ht^2 + [\dot{I}_\perp(0) - \dot{I}_z(0)]t + I_\perp(0) - I_z(0)$, leading to the asymptotics $I_\perp(t) \sim I_z(t) + 4Ht^2$ as $t \to +\infty$. When, e.g., $H<0$, the latter estimate shows that the mean longitudinal size must necessarily diverge faster than $2\sqrt{|H|}t$ as $t \to +\infty$. These results, valid for all initial conditions, improve the ones reported in Refs. [3,7,8], where the absence of collapse in the critical case was definitively established for positive initial rates $\dot{I}_\perp(0) \geq 0$ only.

*(c) Absence of transverse collapse in total compression regimes at the supercritical dimension $D>2/\sigma$.* We now investigate the supercritical dimension $D>2/\sigma$ for wave fields characterized by a transverse distribution defined at the critical dimension number $D_\perp = 2/\sigma$ and an axial one defined for $D_z = 1/\sigma$. We here consider an evolution of the wave field $u(x,t)$ forced in the so-called "total" compression regime. By "total" compression, we mean that both the transverse and longitudinal sizes of the localized wave field are assumed to shrink in space with $\dot{I}_\perp < 0$ and $\dot{I}_z < 0$. Such constraints compress the solution $u(x,t)$ simultaneously in all spatial directions, which could be believed to promote at least a transverse collapse. Paradoxically, we show in the

following that a transverse collapse never occurs in this case. Making use of the inequality (12) in order to bound the various gradient norms, we estimate the virial identities (9) and (10) as follows :

$$\ddot{I}_\perp = 8H + 8\|\nabla_z u\|_2^2 \geq 8H + 2C^2/I_z, \qquad (22)$$

$$\ddot{I}_z = -4H + 4\|\nabla_z u\|_2^2 + 4\|\nabla_\perp u\|_2^2 > -4H + C'^2/I_\perp, \qquad (23)$$

with $C' = D_\perp N$. We then multiply Eq. (22) by $\dot{I}_z < 0$ and Eq. (23) by $\dot{I}_\perp < 0$, add the resulting inequalities, and perform an integration over time to get

$$\dot{I}_z \dot{I}_\perp - 8HI_z - 2C^2 \ln I_z + 4HI_\perp - C'^2 \ln I_\perp \leq \mathcal{E}''(0). \quad (24)$$

Here, $\mathcal{E}''(0) < +\infty$ is the first integral of motion associated with the crossed system (22) and (23) and defined by the LHS of (24) at the initial moment when both $I_\perp$ and $I_z$ are ensured to decrease. Taking next into account that $I_z(t)$ remains bounded from above by $I_z(0)$, by virtue of the requirement $\dot{I}_z < 0$, and from below by the quantity $I_{z,min}$ defined in point ($a$), we deduce from the inequality (24) that $I_\perp(t)$ can never reach zero. Thus, a collapse in the transverse space cannot appear under these conditions, which could be thought to privilege it. This result simply indicates that in a regime where the wave field would self-contract along its longitudinal axis, a complete shrinking of the field distribution in the transverse plane cannot be realized. Consequently, the remaining possibility to promote a transverse collapse lies in the so-called compression-extension regime, where the solution still compresses in the transverse space while it continuously extends in the longitudinal one. From a physical viewpoint, this situation appears in the cubic case $\sigma = 1$ to be the most dangerous for the occurrence of a transverse collapse, since it would amount to considering a wave field stretching along the $z$ axis and therefore ultimately behaving as a 2D waveguide that could undergo a finite-time collapse. In this respect, it is worth mentioning that if a transverse collapse occurred in the 3D case $D_\perp = 2$ and $D_z = 1$, it would cause the divergence of the transverse gradient norm by virtue of the inequality (12), so that the longitudinal gradient norm should necessarily be bounded from below. This property follows from the inequality $\|u\|_4^4 \leq CN^{1/2}\|\nabla_\perp u\|_2^2 \|\nabla_z u\|_2$ obtained by using the Sobolev embedding theorem where $C$ is some positive constant. After a simple rescaling of the gradient norms, the best constant $C_{best}$ can be found by the same way as in [12,13] and it is equal to $C_{best} = 1/N_0$. Here $N_0(=18.94)$ is the mass of the 3D spherical-symmetric ground soliton solution $u_0(|x|)$ of the *elliptic* NSE: $-u_0 + \nabla^2 u_0 + u_0^3 = 0$.

This enables us to show that in the case of a transverse blowup $\|\nabla_\perp u\|_2^2 \to +\infty$ as $t \to t_c$, the constancy of the Hamiltonian

$$H \geq \|\nabla_\perp u\|_2^2 (1 - N^{1/2}\|\nabla_z u\|_2 / 2N_0) - \|\nabla_z u\|_2^2$$

implies that the integral $N$ must exceed the limiting value

$$N \geq 4N_0^2 / \|\nabla_z u\|_2^2. \qquad (25)$$

This criterion can be viewed as a necessary condition for collapse in the sense that the total mass should be greater than some critical value defined by the gradient distribution along $z$. It can be compared to the critical collapse for the 2D ENSE when the collapse is possible as the intensity exceeds some critical value (for details, see, for instance, [12,5]). As announced above, this criterion also demonstrates a bound from below of the gradient norm, but not from *above*, as it should naturally be required to predict the possible occurrence of a transverse collapse from the inequality (22), which remains unsolved at the present state.

In summary, by using estimates constructed from virial-type identities, we have shown that for nonlinearities satisfying (4), solutions to the hyperbolic nonlinear Schrödinger equation will asymptotically stretch along their longitudinal axis. This dynamics is compatible with the property that stationary solutions to HNSE can only be *unlocalized* along the longitudinal direction. Furthermore, for a power-law nonlinearity, we have demonstrated the absence of transverse collapse at the critical dimension. This conclusion applies to supercritical solutions when both of their transverse and longitudinal mean square radius are assumed to self-contract in time. These results, even though they do not strictly rule out the occurrence of a blowup-type singularity, give a strong indication for the absence of collapsing solutions to the HNSE. The question of the occurrence of collapse for a transverse compression accompanied by a longitudinal extension of the wave field, however, remains open.

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